Geometry 4 - Analytic Geomtry

TSS Math Club

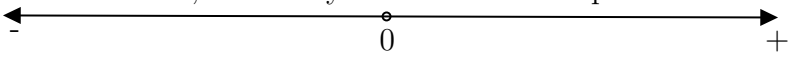
Nov 2022

1 Preliminary

1.1 Real Line

1.1.1 Definition

A number line is a picture of a graduated straight line that serves as visual representation of the real numbers. Every point of a number line is assumed to correspond to a real number, and every real number to a point.



1.2 Ordered Pair

1.2.1 Definition

Informal:

> For any two objects a and b, the ordered pair (a, b) is a notation specifying the two objects a and b, in that order.

Formal:

> (a,b)={{a},{a,b}}

1.2.2 Property

$$(a, b) = (c, d) \iff a = c \wedge b = d$$

1.3 Cartesian Product

1.3.1 Definition

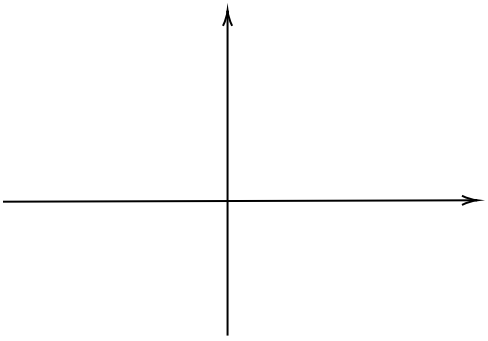
The Cartesian product of two sets A and B, denoted $A \times B$, is the set of all ordered pairs (a, b) where a is in A and b is in B.

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

2 Cartesian Plane

2.1 Definition

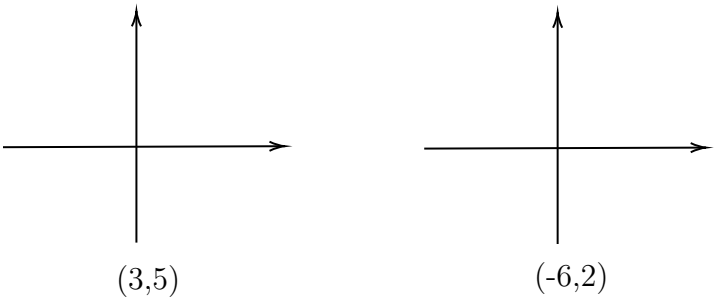
2.2 Visual Representation



2.3 Point

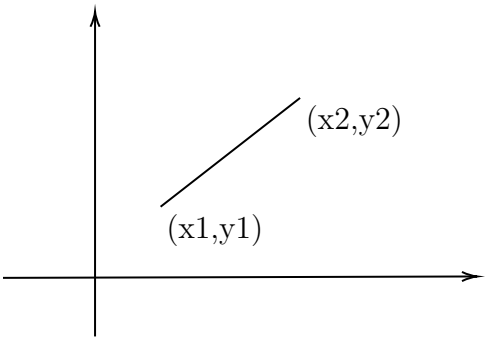
2.3.1 Definition

2.3.2 Plot points



2.4 Metric on the Plane

2.4.1 Distance formula



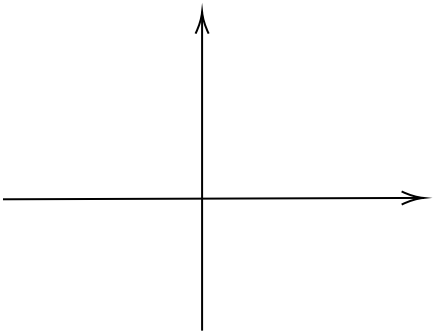
2.4.2 Example

Find the distance between (1,3) and (6,7).

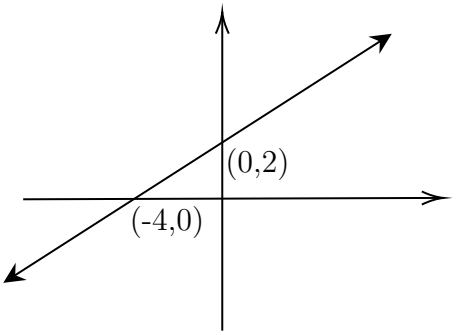
2.5 Line

2.5.1 General Formula

2.5.2 Examples



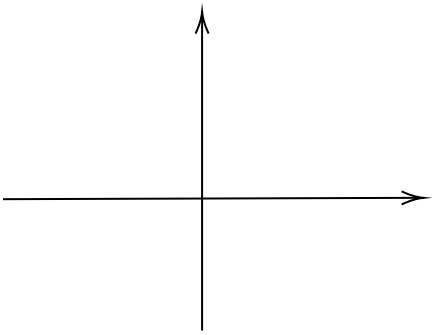
3x+4y+12=0



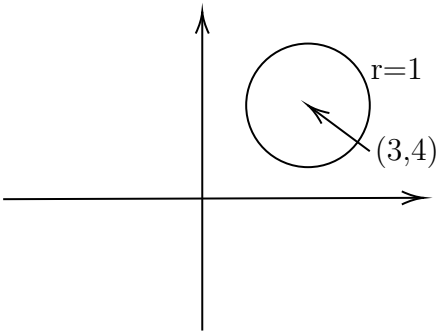
2.6 Circle

2.6.1 General Formula

2.6.2 Examples



x^2+2x+y^2+4x+2=0



2.7 Point to Line Distance Formula

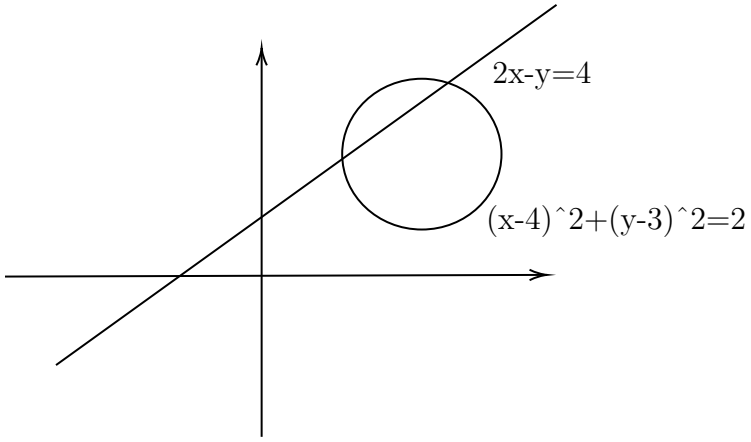
The distance between the line $ax + by + c = 0$ and point (x_1, y_1) is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

2.8 Intersection

2.8.1 How to find intersection between two curve?

2.8.2 Example



2.8.3 Find the Radical Axis of Two Circles

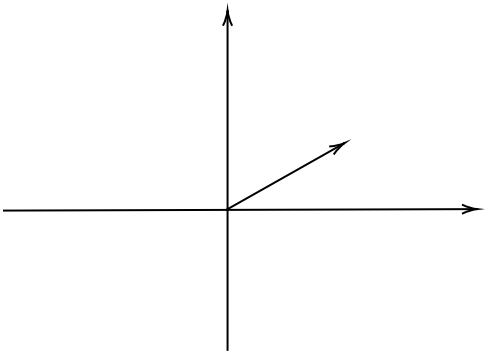
Definition:

Find the radical axis bewteen $x^2 + y^2 = 5$ and $x^2 + 3x + y^2 - 7y + 3 = 0$.

3 Vector

3.1 Definition

3.2 Visual Representation

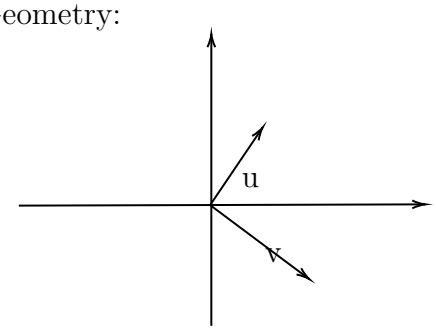


3.3 Addition, Substraction and Scalar Multiplication of Vectors

3.3.1 Addition of Vectors

Algebra:

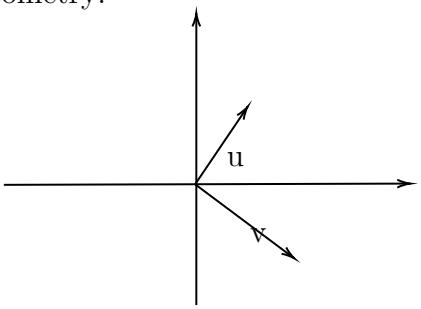
Geometry:



3.3.2 Substraction of Vectors

Algebra:

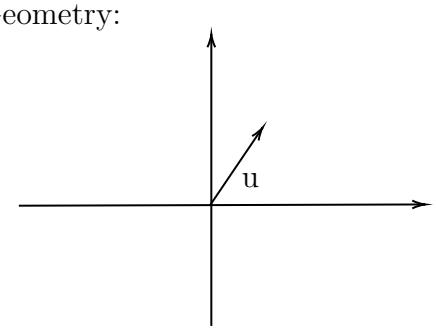
Geometry:



3.3.3 Scalar Multiplication of Vector

Algebra:

Geometry:



3.4 Dot Product

3.4.1 Definition: Dot Product on 2D

If $x = (x_1, x_2)$ and $y = (y_1, y_2)$, then

$$x \cdot y = x_1 y_1 + x_2 y_2$$

3.4.2 Property: Dot Product

- positivity:
- definiteness:
- additivity:
- homogeneity:
- symmetry:

3.4.3 Dot Product and Metric

3.4.4 Penpendicularity

3.4.5 Dot Product and Cosine Law

3.4.6 Dot Product as Projection

3.4.7 Problem (1975 USAMO Q2)

Let A, B, C, D denote four points in space and AB the distance between A and B, and so on. Show that

$$AC^2 + BD^2 + AD^2 + BC^2 \geq AB^2 + CD^2.$$

3.5 Determinant

3.5.1 Definition

3.5.2 Formula

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} =$$

3.5.3 3D Determinant and Area of a Triangle

Definition:

Formula:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ h & i & j \end{vmatrix} = a \begin{vmatrix} e & f \\ i & j \end{vmatrix} - b \begin{vmatrix} d & f \\ h & j \end{vmatrix} + c \begin{vmatrix} d & e \\ h & i \end{vmatrix}$$

Area of a Triangle with Vertex $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is

3.5.4 Shoelace Theorem

Suppose the polygon P has vertices (a_1, b_1), (a_2, b_2), ... , (a_n, b_n), listed in clockwise order. Then the area (A) of P is

$$A = \frac{1}{2} \left| \sum_{i=1}^{n} \det \begin{pmatrix} x_i & x_{i+1} \\ y_i & y_{i+1} \end{pmatrix} \right|$$

Proof

Appendix: Mathematical Induction

Mathematical Induction is a special way of proving things. It has only 2 steps:

- Step 1. Show it is true for the first one
- Step 2. Show that if any one is true then the next one is true

Then all are true

Example

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$