# Geometry 2 - Circles

TSS Math Club

Oct 2022

## 1 Basic property of Circles

### 1.1 Definition of Circles

### 1.2 Terms to describe geometric object related to circles
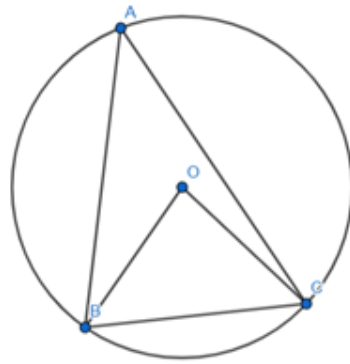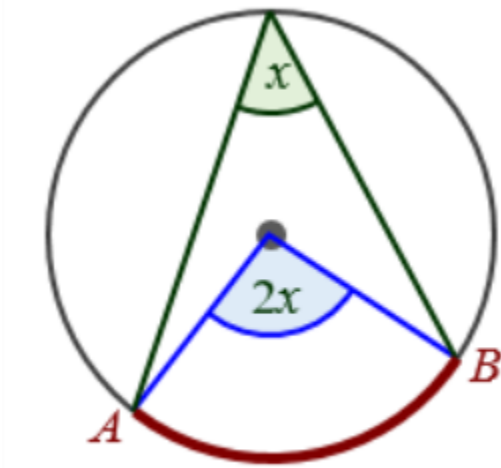
Center:
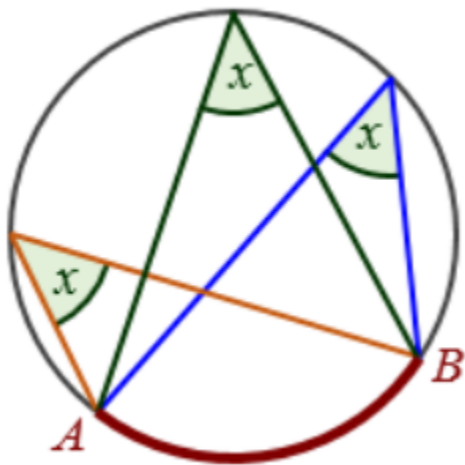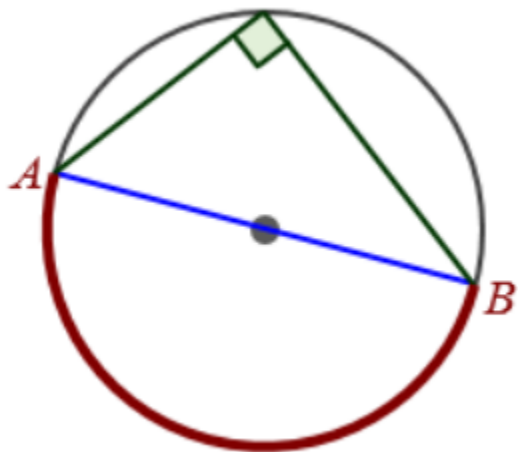
Radius:

Arc:

Chord:

Central angle:

Inscribed angle:



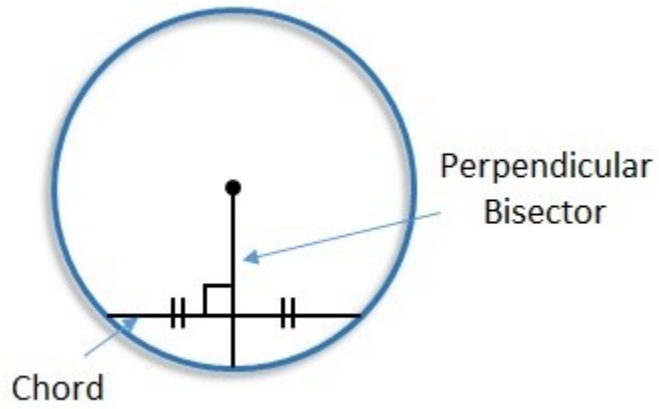### 1.3 Central angle is twice any inscribed angle

## 1.4 Inscribed angles subtended by the same arc are equal

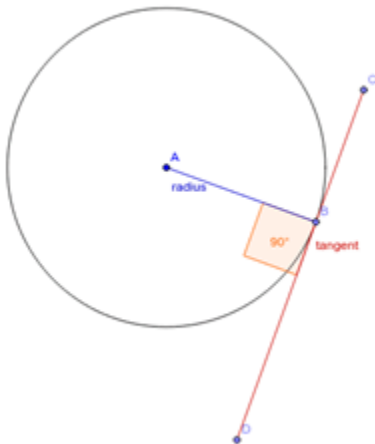## 1.5 Angle subtended by a diameter is $90°$

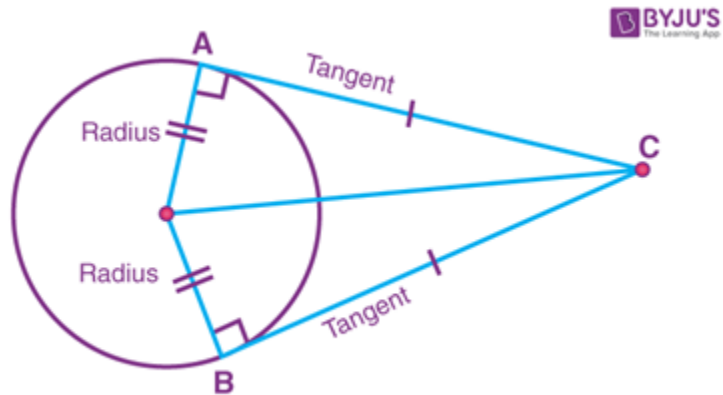## 1.6  Perpendicular chord theorem

Perpendicular
Bisector

Chord

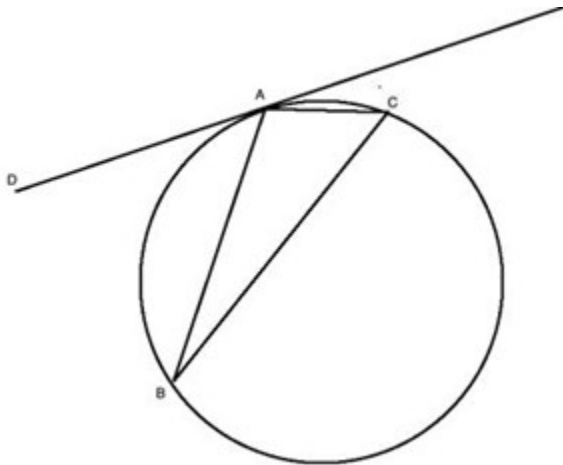## 1.7  Tangent to a circle

### 1.7.1  Definition:

### 1.7.2  The radius from the center of the circle to the point of tangency is perpendicular to the tangent line

### 1.7.3  The length of tangents from a point to a circle are equal
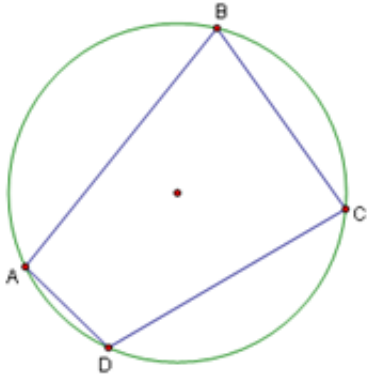


### 1.7.4  Tangent-Chord Theorem: the angle formed between a chord and a tangent line to a circle is equal to the inscribed angle on the other side of the chord

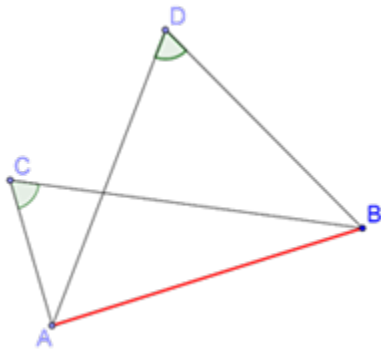# 2 Cyclic Quadrilateral (Four points cyclic)

## 2.1 Definition

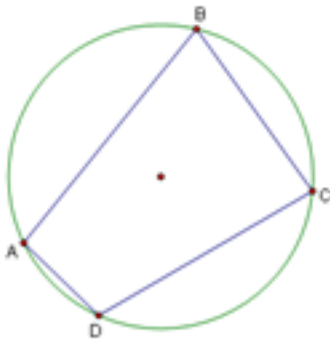## 2.2 Opposite angles are added up to $180°$



## 2.3 How to prove four points cyclic

### 2.3.1 Prove these four points lies equally distance to another point — the center of the circle

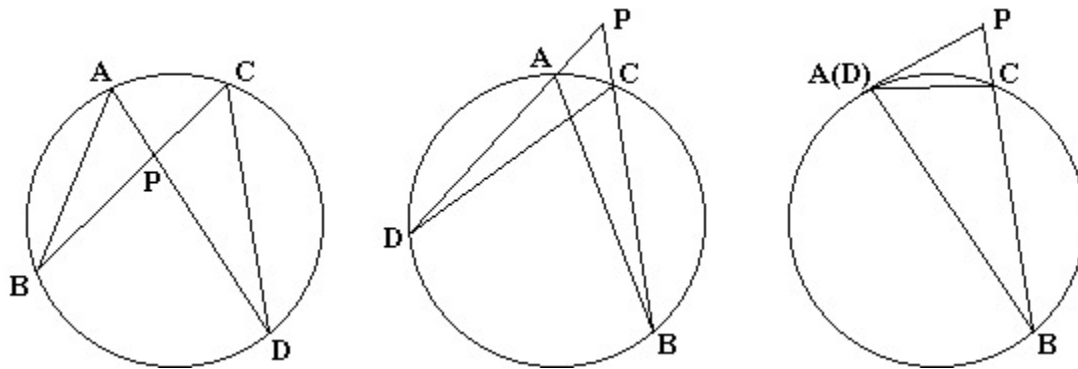### 2.3.2 Two equal angles subtend a segment (chord in the circle)



### 2.3.3 Opposite angles are added up to $180°$
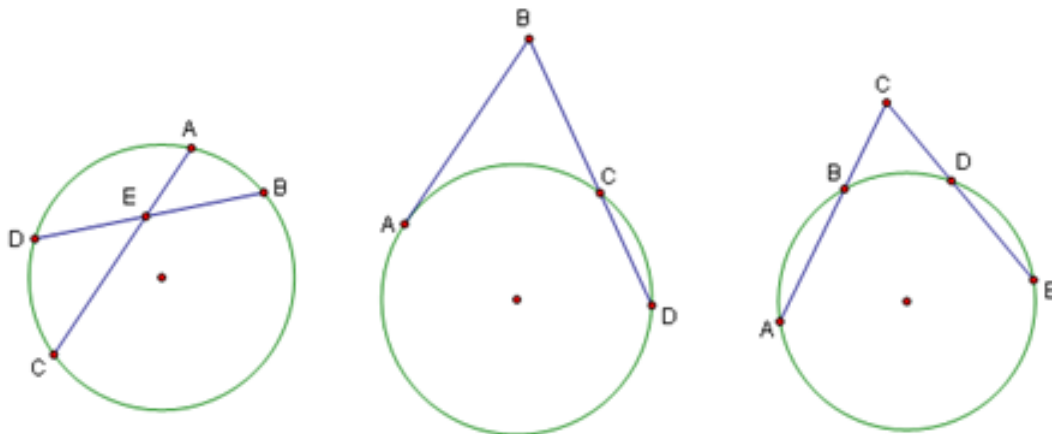
# 3  Similar triangles involving a circle

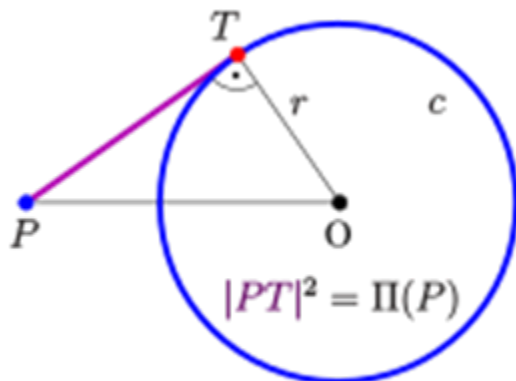## 3.1  Identify as many similar triangles as possible



## 3.2  Power of a point

### 3.2.1  Definition:

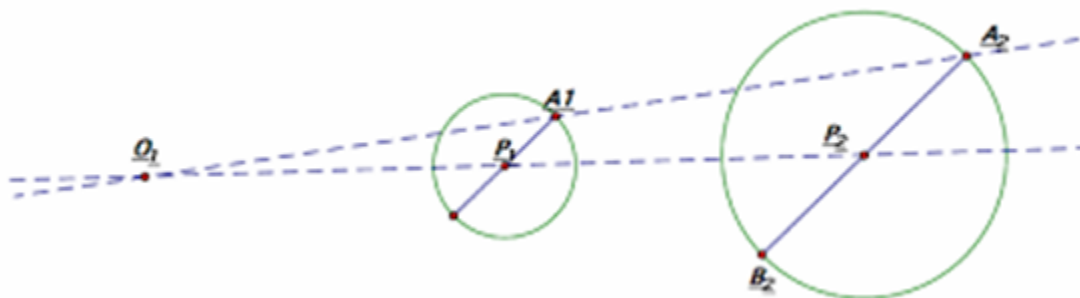### 3.2.2  Power of point is fixed regardless the choice of chord

### 3.2.3 Power of a point formula

$$PoP = PO^2 + r^2$$



## 3.3 Homothety involving circles

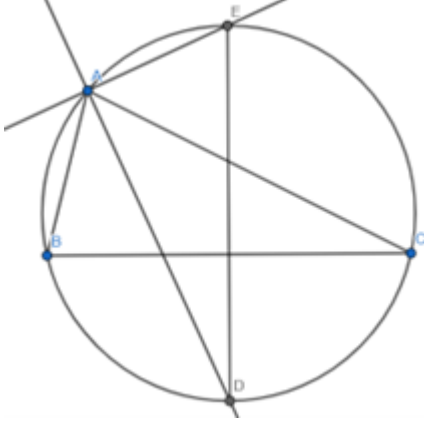### 3.3.1 Homothety of a circle is a circle



### 3.3.2 Ratios in the homothety

# 4 Problems

## 4.1 Problem

Given AD AE are the internal, external angle bisector of angle A, such that D,E are the intersection of the angle bisectors with the circumcircle. Prove DE is a diameter of the circle.
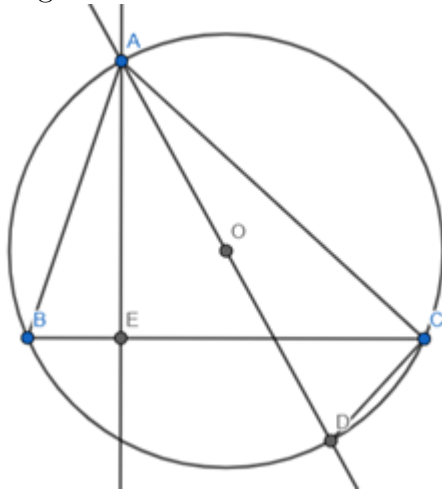
## 4.2 Problem

Given Circle C1, C2 intersect at A, B, CD is the common tangent to both circles, E is the intersection of AB and CD. Prove E is the midpoint of CD.
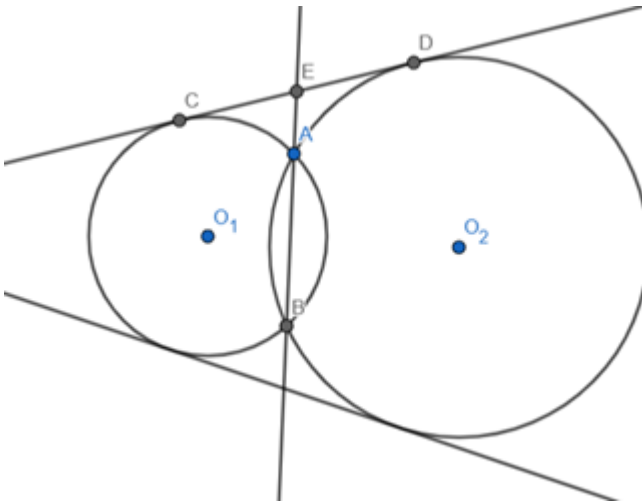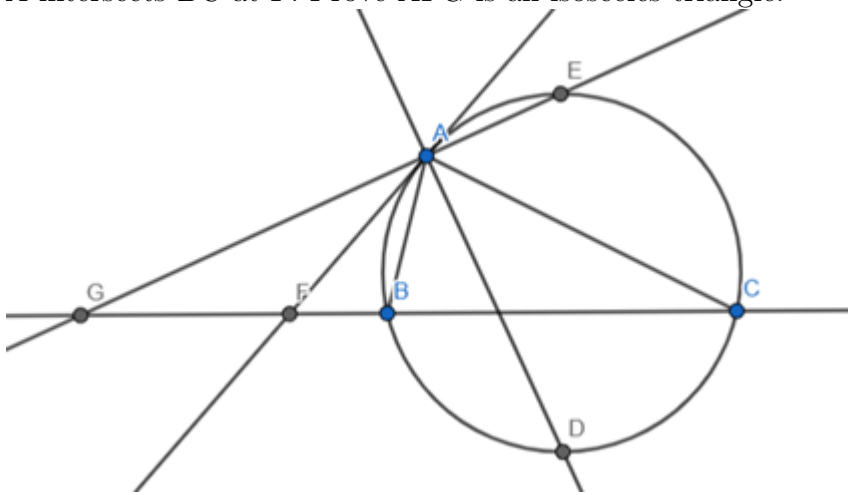
## 4.3 Theorem

In a triangle abc=4RS

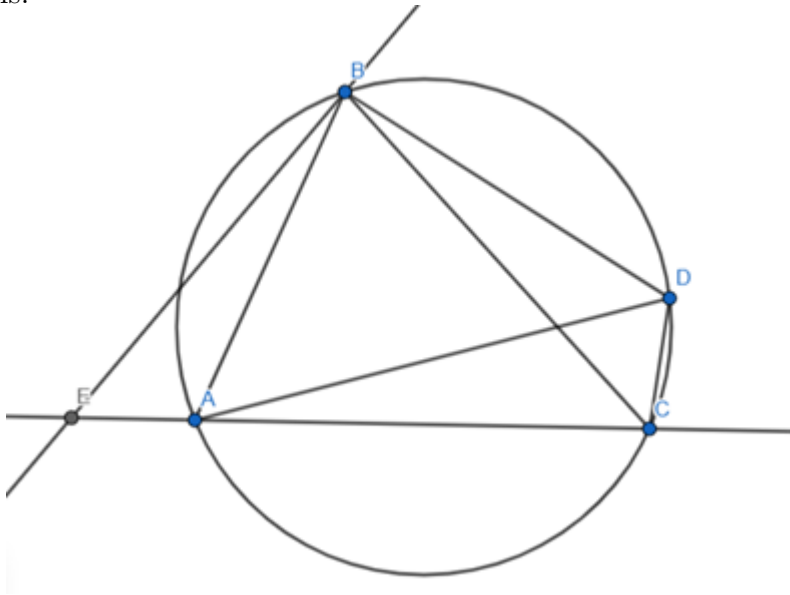## 4.4 Problem

Given AE is the external angle bisector of angle A, AE intersects BC at G, the tangent at A intersects BC at F. Prove AFG is an isosceles triangle.

## 4.5 Ptolemy's theorem

If a quadrilateral is inscribable in a circle then the product of the lengths of its diagonals is equal to the sum of the products of the lengths of the pairs of opposite sides. Or ab+cd=xy where a,b,c,d are the sides of the quadrilateral and x,y are the diagonals.

## 4.6  Problem

In $\triangle ABC$, point D is inside of ABC such that $\angle DAC = \angle DCA = 30°$ and $\angle DBA = 60°$. E is the midpoint on BC and F is a trisect point on AC such that $CF = \frac{CA}{3}$. Prove $DE \perp EF$.