Geometry 1 - Basis of Geometry

TSS Math Club

Oct 2022

1 Parallelism and basic geometry

1.1 Parallel lines

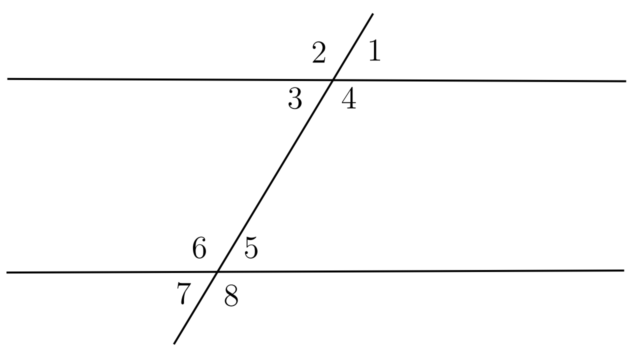
1.1.1 Parallel postulate

If a line segment intersects two straight lines forming two interior angles on the same side that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

1.1.2 Definition

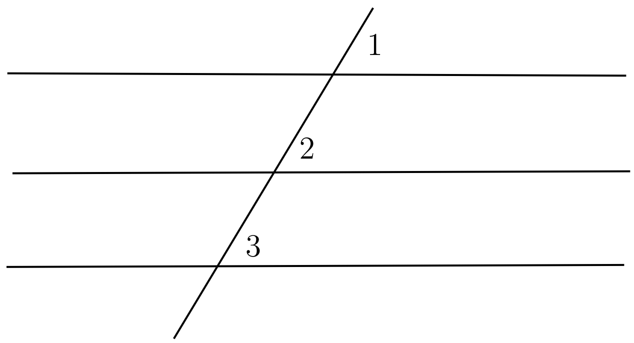
Two lines in two-dimensional Euclidean space are said to be parallel if they do not intersect.

1.1.3 Parallel Lines and Transversal

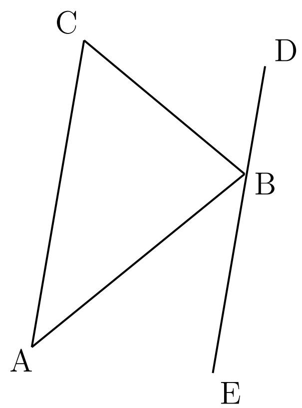


- Corresponding angles are equal ($\angle 1 = \angle 5$)
- Alternate angles are equal ($\angle 3 = \angle 5$)
- Co-interior angles are supplementary to each other ($\angle 4 + \angle 5$=180°)

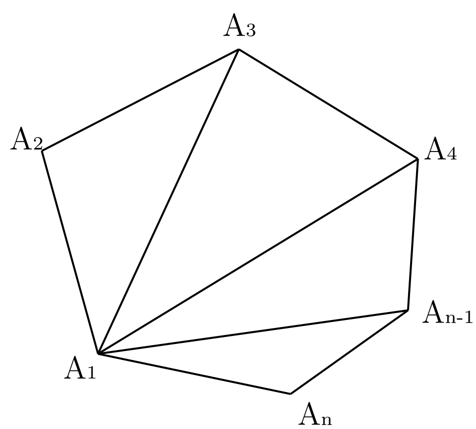
1.2 Parallel is transitive



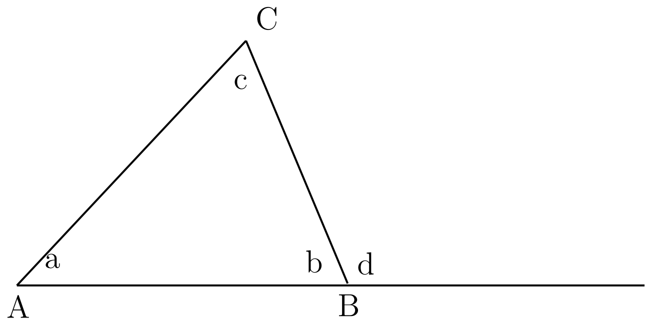
1.3 Sum of the interior angles of a triangle is 180°



1.4 Sum of interior angles of a n-gon is $(n-2)180^\circ$

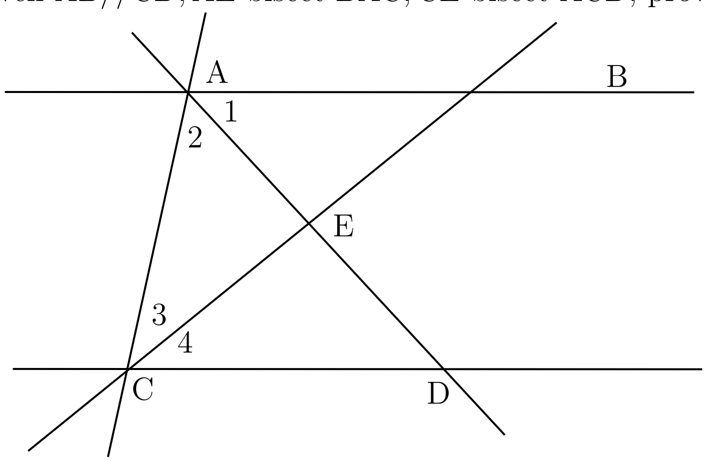


1.5 Exterior angle theorem



1.6 Another problem

Given AB//CD, AE bisect BAC, CE bisect ACD, prove AE perpendicular to CE.



2 Congruence

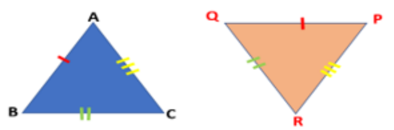
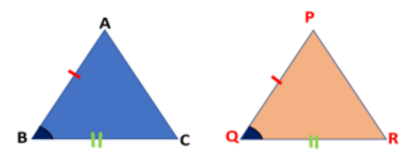
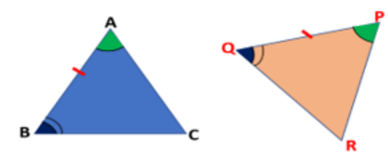
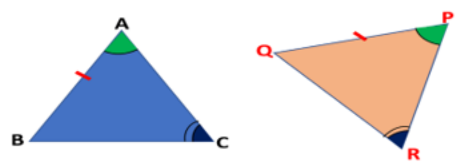
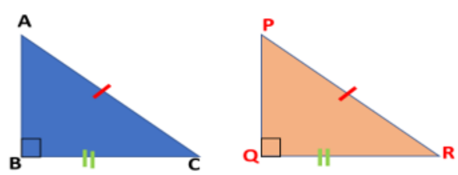
2.1 Definition

The same shape and size.

2.2 Method to prove congruence

2.2.1 5 Methods:

Congruent Triangles

SSS (Side – Side – Side) Congruency		*When the three sides of a triangles are equal to the other three sides of another triangle.* $\Delta ABC \cong \Delta PQR$
SAS (Side – Angle – Side) Congruency		*When two sides and the included angle on one triangle is equal to the two sides and included angle of another triangle.* $\Delta ABC \cong \Delta PQR$
ASA (Angle – Side – Angle) Congruency		*When two angles and the included side of one triangle are equal to two angles and the included side of another triangle.* $\Delta ABC \cong \Delta PQR$
AAS (Angle – Angle – Side) Congruency		*When two angles and the non-included side of one triangle are equal to two angles and the non-included side of another triangle.* $\Delta ABC \cong \Delta PQR$
RHS (Right Angle – Hypotenuse – Side) Congruency		*When hypotenuse and one side of a right triangle is equal to the hypotenuse and other side of another right triangle.* $\Delta ABC \cong \Delta PQR$

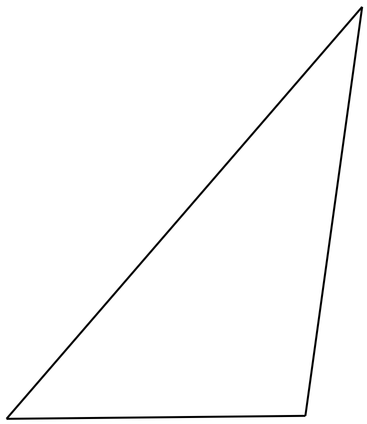
2.2.2 How to write in a contest

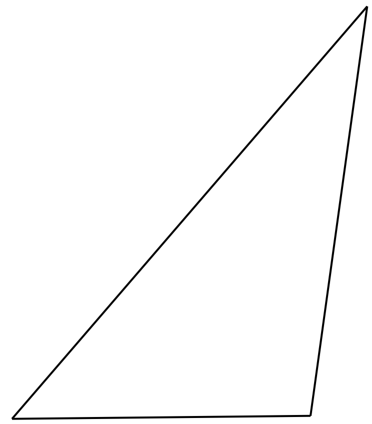
In the $\triangle \mathrm{ABC}$ and $\triangle \mathrm{A'B'C'}$

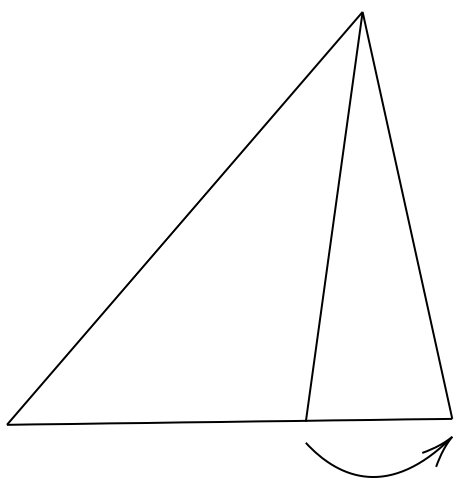
$$\left\{\begin{array}{l}\text{Condition 1}\\ \text{Condition 2}\\ \text{Condition 3}\end{array}\right.$$

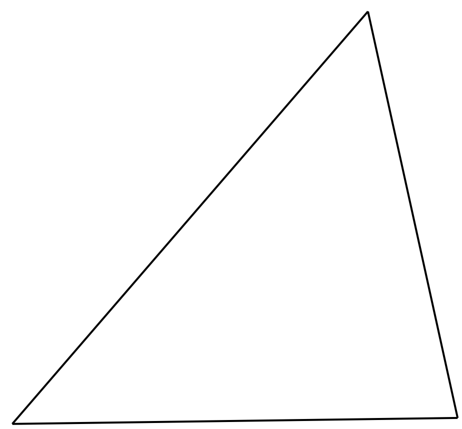
$$\therefore \triangle \mathrm{ABC} \cong \triangle \mathrm{A'B'C'}$$

2.3 Side-Side-Angle (SSA), the ambiguous case



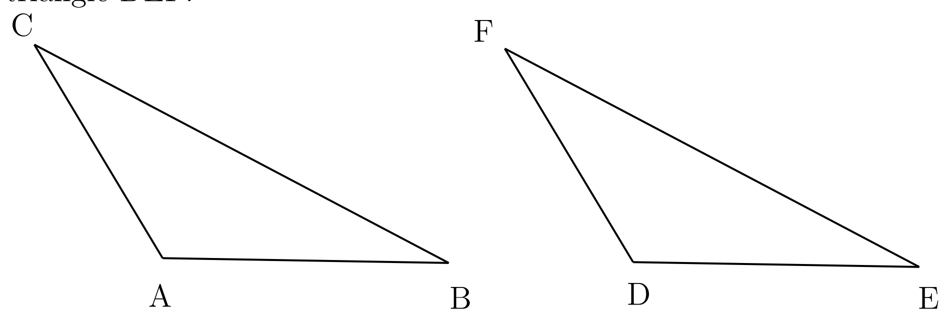






2.3.1 Proof related to the ambiguous case

Given AB = DE, BC = EF, angle A = angle D > 90°, prove triangle ABC congruent to triangle DEF.



2.4 Useful congruencies

- Rotation
- Translation

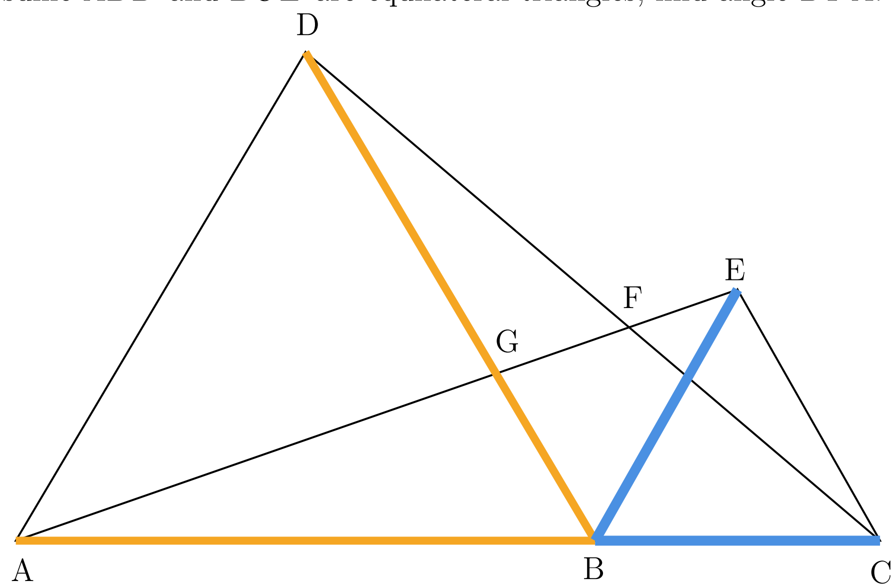
2.5 Properties of congruence

- Equal side length
- Equal angles

2.6 Problems

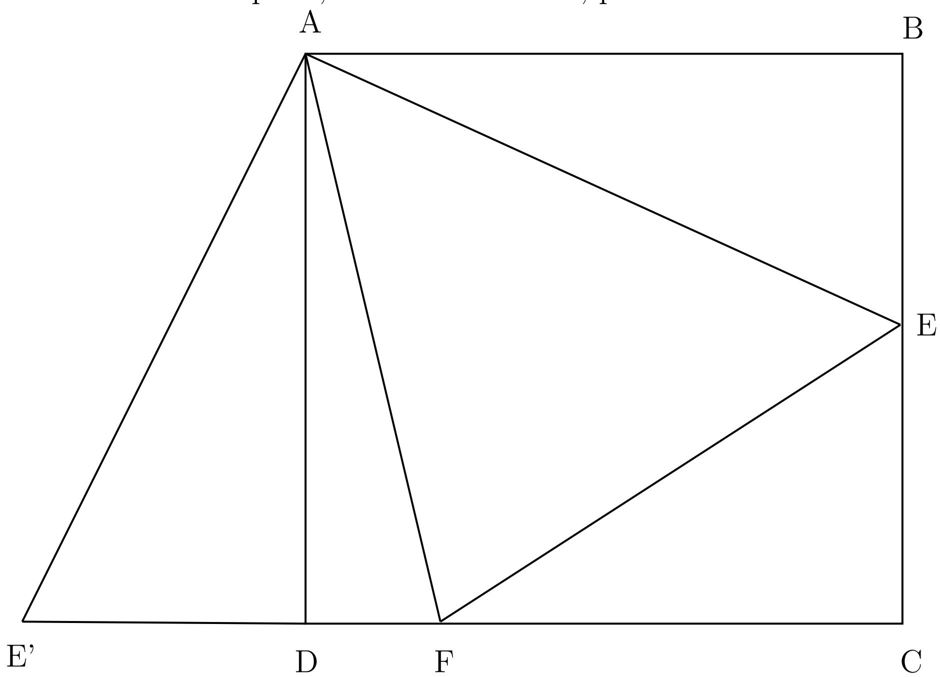
2.6.1 Problem

Assume ABD and BCE are equilateral triangles, find angle DFA.



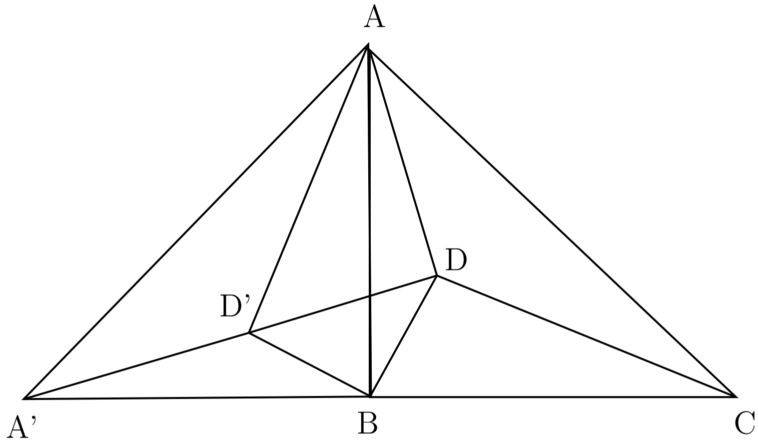
2.6.2 Problem

Given $ABCD$ is a square, and $\angle EAF = 45°$, prove AE bisect BEF.



2.6.3 Problem

Given $AB = BC$ and angle $ABC = 90^\circ$, $AD = \sqrt{5}$, $BD = \sqrt{2}$, $DC = 3$, find angle ADB.

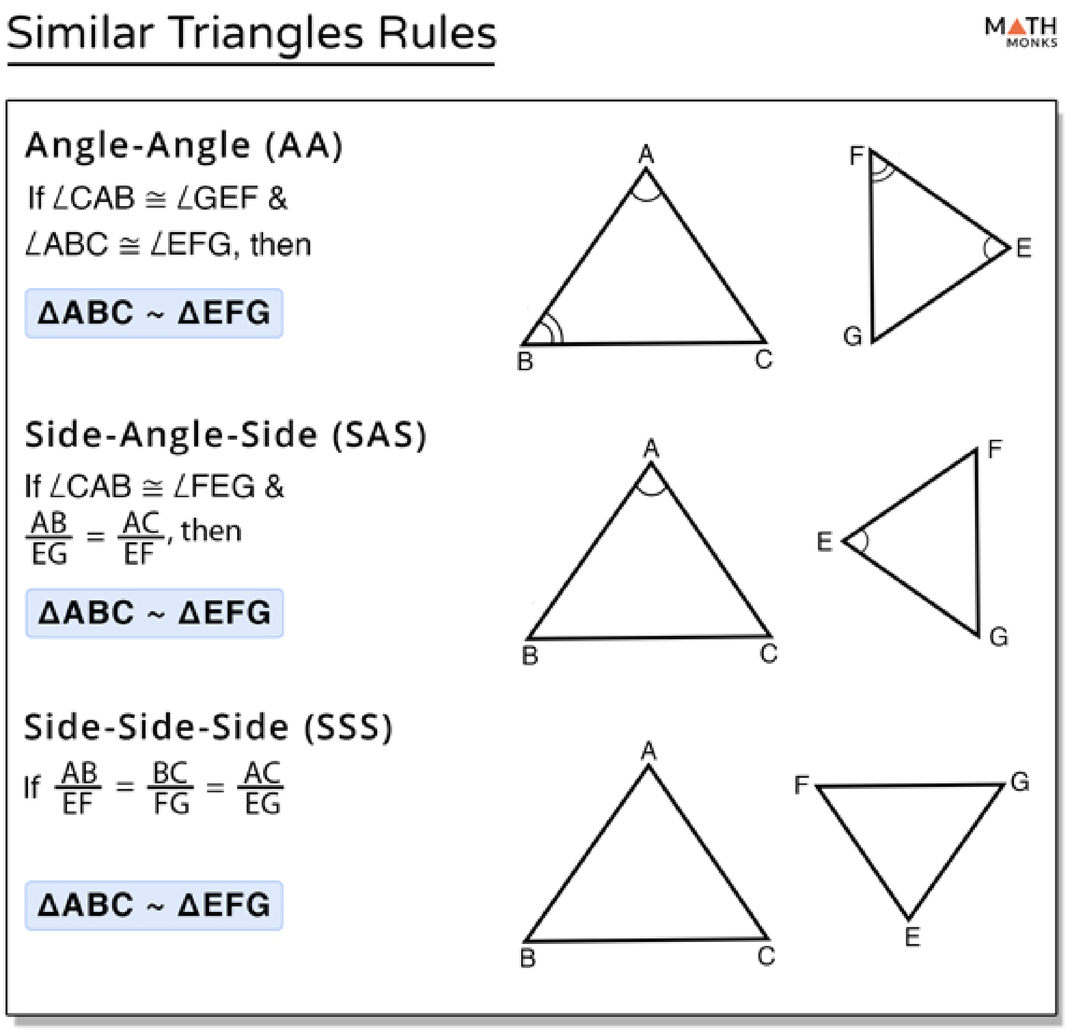


3 Similarity

3.1 Definition

In Euclidean geometry, two objects are similar if they have the same shape, or one has the same shape as the mirror image of the other.

3.2 Method to Prove similar triangles

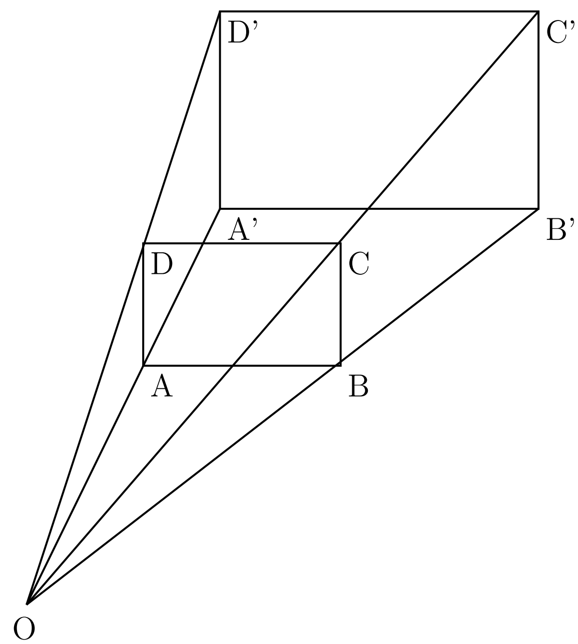


3.3 Properties of similar triangles

- Ratio of corresponding sides
- Equal angles

4 Homothety

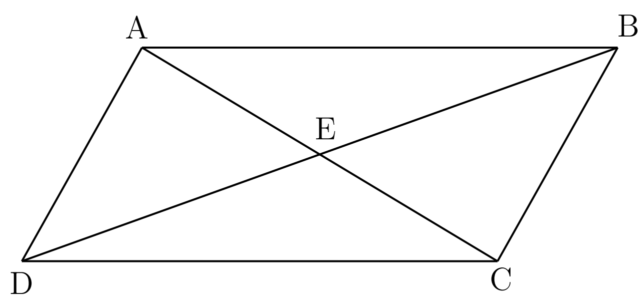
4.1 Definition



4.2 Properties

5 Quadrilaterals

5.1 Parallelogram



5.1.1 Definition

5.1.2 Properties

5.2 Rhombus

5.2.1 Definition

5.2.2 Properties

5.3 Rectangle

5.3.1 Definition

5.3.2 Properties

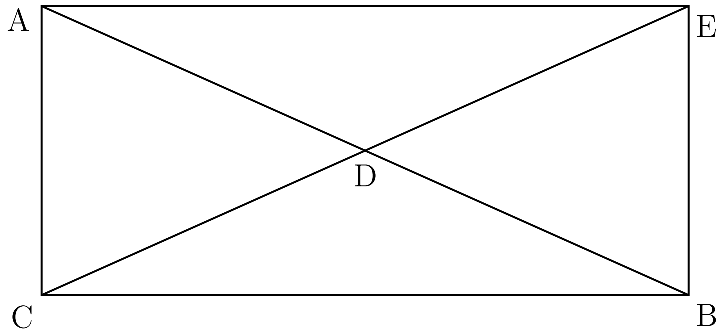
5.4 Square

5.4.1 Definition

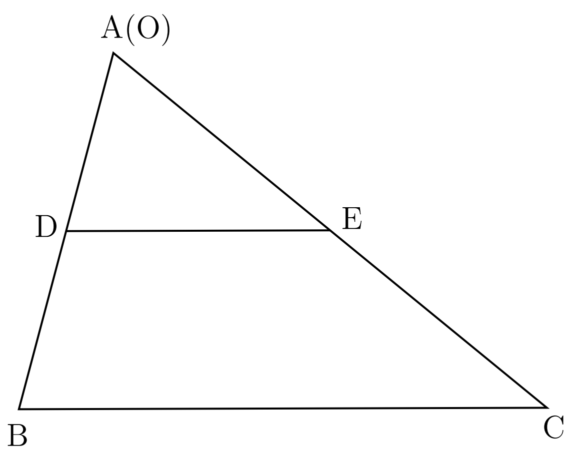
5.4.2 Properties

6 Midpoint

6.1 Midpoint of a right triangle



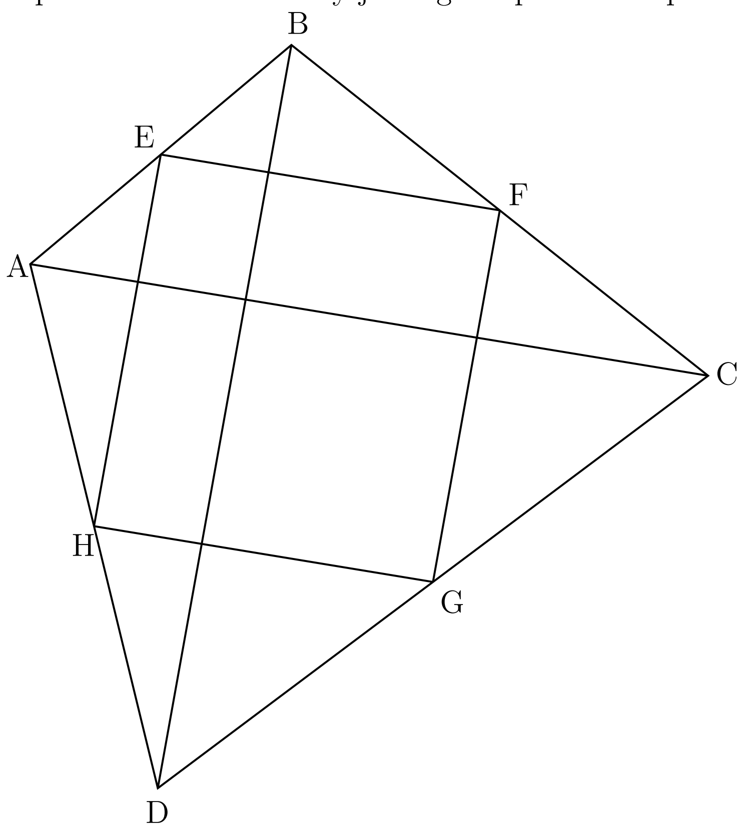
6.2 Midsegment



7 Problems

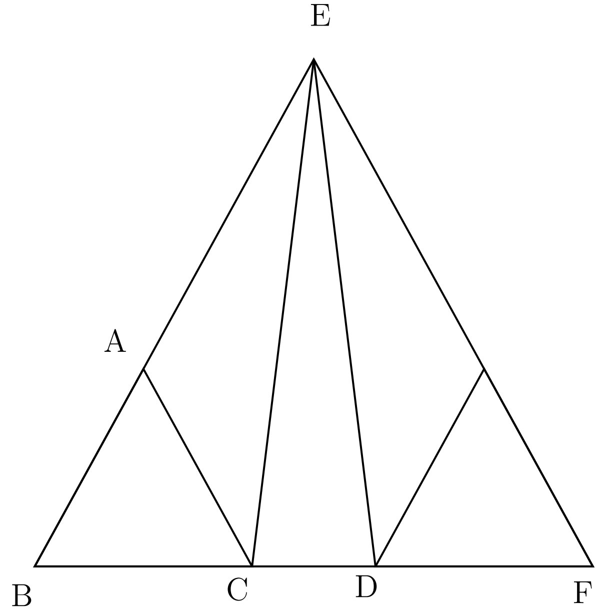
7.1 Problem

The quadrilateral formed by joining midpoint of a quadrilateral is a parallelogram.



7.2 Problem

Given equilateral $\triangle ABC$, extend BC to D and BA to E to let $AE = BD$. Join CE and DE. Prove $\triangle ECD = \triangle EDC$.



7.3 Problem

In quadrilateral $ABCD$ with $AD = BC$, E and F are midpoints on AB and CD, respectively. EF meets AD at G, meets BC at H. Prove $\angle DGF = \angle CHF$.